# Chapter 9
## PROBABILITY

### EXERCISE 9A.1

**1** **a** P(inside a square) $= \frac{113}{145} \approx 0.78$ **b** P(on a line) $= \frac{32}{145} \approx 0.22$

**2** Total frequency $= 17 + 38 + 19 + 4 = 78$

**a** P(20 to 39 seconds) $= \frac{38}{78} \approx 0.487$ **b** P($> 60$ seconds) $= \frac{4}{78} \approx 0.051$

**c** P(between 20 and 59 seconds inclusive) $= \dfrac{38 + 19}{78} \approx 0.731$

**3**

| *Calls/day* | *No. of days* |
|---|---|
| 0 | 2 |
| 1 | 7 |
| 2 | 11 |
| 3 | 8 |
| 4 | 7 |
| 5 | 4 |
| 6 | 3 |
| 7 | 0 |
| 8 | 1 |

**a** Survey lasted $2 + 7 + 11 + 8 + 7 + 4 + 3 + 0 + 1$ $= 43$ days

**b** **i** P(0 calls) $\approx \dfrac{2}{43}$ $\approx 0.0465$

**ii** P($\geqslant 5$ calls) $\approx \dfrac{4 + 3 + 0 + 1}{43}$ $\approx 0.186$

**iii** P($< 3$ calls) $\approx \dfrac{2 + 7 + 11}{43}$ $\approx 0.465$

**4** Total frequency $= 37 + 81 + 48 + 17 + 6 + 1$ $= 190$

**a** P(4 days gap) $\approx \dfrac{17}{190}$ $\approx 0.0895$

**b** P(at least 4 days gap) $\approx \dfrac{17 + 6 + 1}{190}$ $\approx 0.126$

### EXERCISE 9A.2

**1** **a** P(female at **C** is a smoker) $= \frac{13}{49} \approx 0.265$

**b** At school **E**, there are $40 + 39 = 79$ 15 year old students.
$7 + 4 = 11$ of these smoke, so $79 - 11 = 68$ do not smoke.
$\therefore$ P(student at **E** is *not* a smoker) $= \frac{68}{79} \approx 0.861$

**c** P(he or she is a smoker) $= \dfrac{48 + 44}{201 + 214} \approx 0.222$

**2** **a** P(complaint in 2008/09 about customer service) $= \frac{1181}{8085} \approx 0.146$

**b** P(complaint about billing) $= \dfrac{1822 + 2102 + 3136 + 3582}{3015 + 4282 + 8085 + 9109} \approx 0.435$

**c** P(not related to either billing or faults in 2009/10)
$= \dfrac{1612 + 836 + 136 + 1940 + 248 + 60 + 311}{9109} \approx 0.565$

**3** **a** **i** P(Feb day in Auburn is $\geqslant 35°$C) $= \frac{5.3}{28} \approx 0.189$

**ii** P(Feb day in Auburn is $< 30°$C) $= \dfrac{28 - 12.6}{28} = 0.55$

**b** P(Temperature $\geqslant 30°$C on a summer day) $= \dfrac{9.4 + 12.3 + 12.6}{31 + 31 + 28} \approx 0.381$

**c** P(It is Jan given it is a $40°$C day) $= \dfrac{1.2}{0.3 + 1.2 + 0.7} \approx 0.545$

## EXERCISE 9B

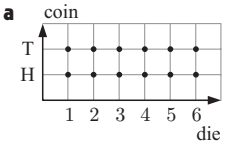
**1** **a** {A, B, C, D}

**b** Let B denote 'a boy' and G denote 'a girl'. {BB, BG, GB, GG}

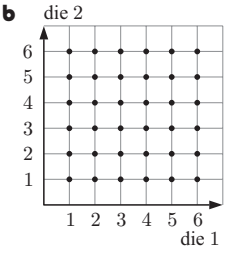
**c** {ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, BACD, BADC, BCAD, BCDA, BDAC, BDCA, CABD, CADB, CBAD, CBDA, CDAB, CDBA, DABC, DACB, DBAC, DBCA, DCAB, DCBA}

**d** Let B denote 'a boy' and G denote 'a girl'. {BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG}

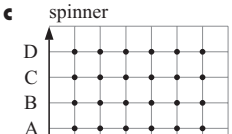
**2** **a**



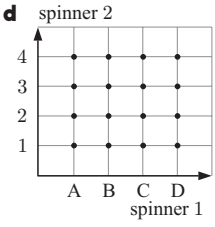

**b**



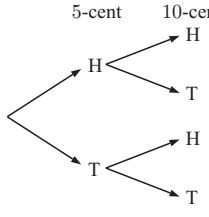

**c**



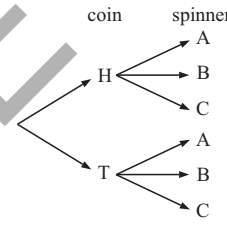

**d**



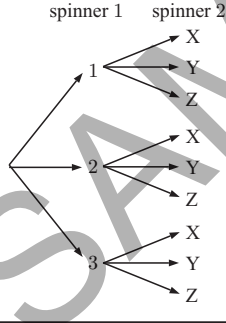

**3** **a** Let H denote 'heads' and T denote 'tails'.



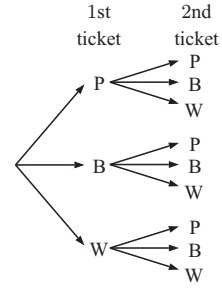

**b** Let H denote 'heads' and T denote 'tails'.




**c**




**d** Let P denote 'pink', B denote 'blue', and W denote 'white'.






## EXERCISE 9C.1

**1** Total number of marbles $= 5 + 3 + 7 = 15$

**a** $P(\text{red}) = \frac{3}{15} = \frac{1}{5}$

**b** $P(\text{green}) = \frac{5}{15} = \frac{1}{3}$

**c** $P(\text{blue}) = \frac{7}{15}$

**d** $P(\text{not red}) = \dfrac{5+7}{15} = \frac{12}{15} = \frac{4}{5}$

**e** $P(\text{neither green nor blue}) = P(\text{red}) = \frac{3}{15} = \frac{1}{5}$

**f** $P(\text{green or red}) = \dfrac{5+3}{15} = \frac{8}{15}$

**2** **a** 8 are brown and so 4 are white.

**b** **i** $P(\text{brown}) = \frac{8}{12} = \frac{2}{3}$ **ii** $P(\text{white}) = \frac{4}{12} = \frac{1}{3}$

**3** **a** P(multiple of 4)
$= P(4, 8, 12, 16, 20, 24, 28, 32, 36)$
$= \frac{9}{36}$
$= \frac{1}{4}$

**b** P(between 6 and 9 inclusive)
$= P(6, 7, 8, \text{or } 9)$
$= \frac{4}{36}$
$= \frac{1}{9}$

**c** $P(> 20)$
$= P(21, 22, 23, 24, ...., 35, 36)$
$= \frac{36 - 20}{36}$
$= \frac{16}{36}$
$= \frac{4}{9}$

**d** $P(9) = \frac{1}{36}$

**e** $P(\text{multiple of } 13)$
$= P(13 \text{ or } 26)$
$= \frac{2}{36}$
$= \frac{1}{18}$

**f** $P(\text{odd multiple of } 3)$
$= P(3, 9, 15, 21, 27, \text{or } 33)$
$= \frac{6}{36}$
$= \frac{1}{6}$

**g** $P(\text{multiple of 4 and 6})$
$= P(\text{multiple of } 12)$
$= P(12, 24, 36)$
$= \frac{3}{36}$
$= \frac{1}{12}$

**h** $P(\text{multiple of 4 or 6})$
$= P(4, 6, 8, 12, 16, 18, 20, 24, 28, 30, 32, 36)$
$= \frac{12}{36}$
$= \frac{1}{3}$

**4** **a** $P(\text{on a Tuesday}) = \frac{1}{7}$

**b** $P(\text{on a weekend}) = \frac{2}{7}$

**c** $P(\text{in July})$
$= \frac{4 \times 31}{365 \times 3 + 366}$ {over 4 year period}
$= \frac{124}{1461}$

**d** $P(\text{in January or February})$
$= \frac{4 \times 31 + 3 \times 28 + 1 \times 29}{3 \times 365 + 1 \times 366}$
{over 4 year period, remember leap years}
$= \frac{237}{1461} = \frac{79}{487}$

**5** **a** Let A denote Antti, K denote Kai, and N denote Neda.
Possible orders are: {AKN, ANK, KAN, KNA, NAK, NKA}

**b** **i** $P(\text{A in middle}) = \frac{2}{6} = \frac{1}{3}$

**ii** $P(\text{A at left end}) = \frac{2}{6} = \frac{1}{3}$

**iii** $P(\text{A not at right end}) = 1 - \frac{1}{3} = \frac{2}{3}$

**iv** $P(\text{K and N are together}) = \frac{4}{6} = \frac{2}{3}$

**6** Let G denote 'a girl' and B denote 'a boy'.

**a** Possible orders are: {BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG}

**b** **i** $P(\text{all boys}) = P(\text{BBB}) = \frac{1}{8}$

**ii** $P(\text{all girls}) = P(\text{GGG}) = \frac{1}{8}$

**iii** $P(\text{boy, then girl, then girl})$
$= P(\text{BGG})$
$= \frac{1}{8}$

**iv** $P(\text{2 girls and a boy})$
$= P(\text{BGG or GBG or GGB})$
$= \frac{3}{8}$

**v** $P(\text{girl is eldest})$
$= P(\text{GBB or GBG or GGB or GGG})$
$= \frac{4}{8} = \frac{1}{2}$

**vi** $P(\text{at least one boy})$
$= \frac{7}{8}$ {all except GGG}

**7** **a** {ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, BACD, BADC, BCAD, BCDA, BDAC, BDCA, CABD, CADB, CBAD, CBDA, CDAB, CDBA, DABC, DACB, DBAC, DBCA, DCAB, DCBA}

**b** **i** $P(\text{A sits on one end}) = \frac{12}{24} = \frac{1}{2}$

**ii** $P(\text{B sits on one of the two middle seats}) = \frac{12}{24} = \frac{1}{2}$

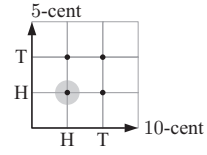
**iii** $P(\text{A and B are together}) = \frac{12}{24} = \frac{1}{2}$

**iv** $P(\text{A, B, and C are together}) = \frac{12}{24} = \frac{1}{2}$

## EXERCISE 9C.2

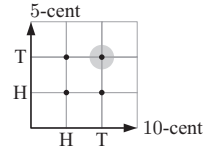
**1** Let H denote 'heads' and T denote 'tails'.

**a**



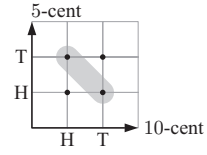

$P(2 \text{ heads}) = \frac{1}{4}$

**b**



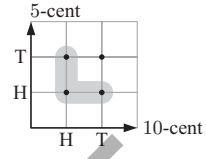

$P(2 \text{ tails}) = \frac{1}{4}$

**c**



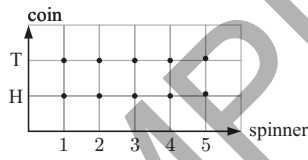

$P(\text{exactly 1 head}) = P(\text{HT or TH})$
$= \frac{2}{4} = \frac{1}{2}$

**d**



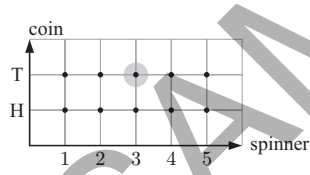

$P(\text{at least 1 head}) = P(\text{HT or TH or HH})$
$= \frac{3}{4}$

**2** **a** Let H denote 'heads' and T denote 'tails'.



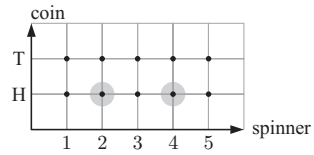

**b** There are $2 \times 5 = 10$ possible combined outcomes.

**c** **i**



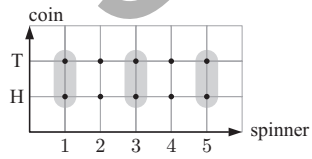

$P(\text{T and 3}) = \frac{1}{10}$

**ii**



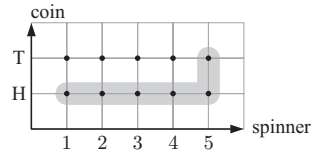

$P(\text{H and even}) = P(\text{H2 or H4})$
$= \frac{2}{10} = \frac{1}{5}$

**iii**



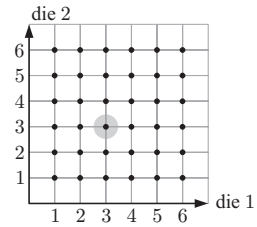

$P(\text{an odd}) = \frac{6}{10} = \frac{3}{5}$

**iv**



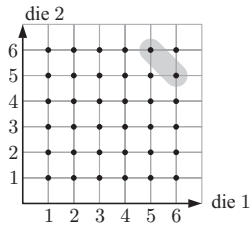

$P(\text{H or 5}) = \frac{6}{10} = \frac{3}{5}$

**3** **a**



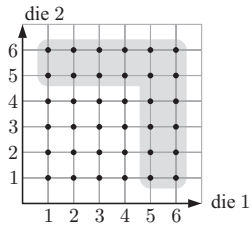

$P(\text{two 3s}) = \frac{1}{36}$

**b**



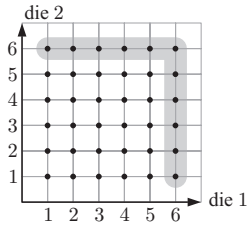

$P(\text{5 and a 6}) = \frac{2}{36} = \frac{1}{18}$

**c**



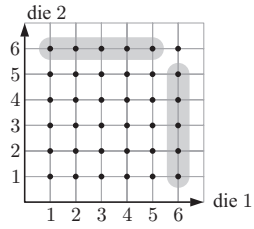

$P(\text{5 or a 6}) = \frac{20}{36} = \frac{5}{9}$

**d**



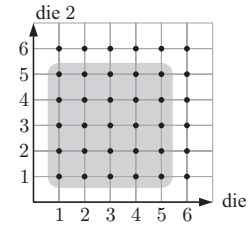

P(at least one 6) $= \frac{11}{36}$

**e**




P(exactly one 6) $= \frac{10}{36} = \frac{5}{18}$

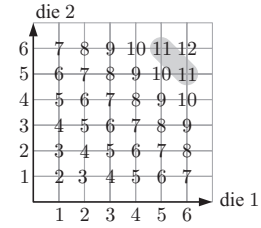
**f**




P(no sixes) $= \frac{25}{36}$

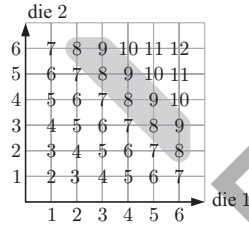
## EXERCISE 9C.3

**1 a, b** **i**



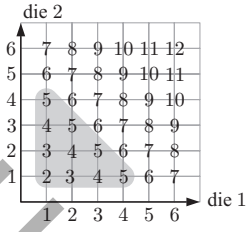

P(sum is 11) $= \frac{2}{36}$
$= \frac{1}{18}$

**ii**



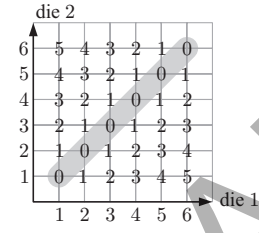

P(sum is 8 or 9) $= \frac{9}{36}$
$= \frac{1}{4}$

**iii**



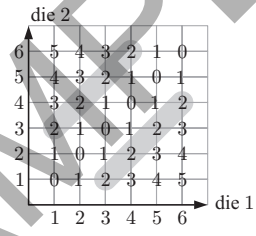

P(sum $< 6$) $= \frac{10}{36}$
$= \frac{5}{18}$

**2 a, b** **i**



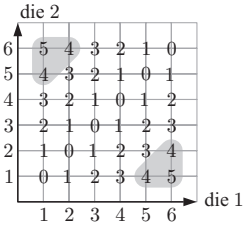

P(result is 0) $= \frac{6}{36}$
$= \frac{1}{6}$

**ii**



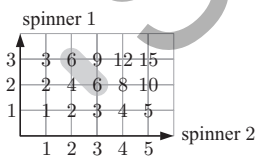

P(result is 2) $= \frac{8}{36}$
$= \frac{2}{9}$

**iii**



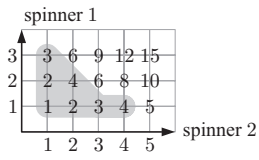

P(result is $> 3$) $= \frac{6}{36}$
$= \frac{1}{6}$

**3 a, b** **i**



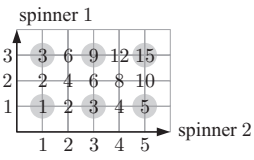

P(result is 6) $= \frac{2}{15}$

**ii**




P(result is $< 5$) $= \frac{7}{15}$

**iii**




P(result is odd) $= \frac{6}{15}$
$= \frac{2}{5}$

## EXERCISE 9D.1

**1 a** P(Rob rolls 4 and Kerry rolls 2) $=$ P(4) $\times$ P(2) {events are independent}
$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

**b** P(Rob rolls odd and Kerry rolls $> 4$)
$=$ P(odd) $\times$ P($> 4$)
$=$ P(1, 3, or 5) $\times$ P(5 or 6)
$= \frac{3}{6} \times \frac{2}{6} = \frac{1}{6}$

**c** P(both roll $> 1$) $=$ P($> 1$) $\times$ P($> 1$)
$= \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

**2** **a** P(H, then H, then H)
$= \text{P(H)} \times \text{P(H)} \times \text{P(H)}$
$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$
$= \frac{1}{8}$

**b** P(T, then H, then T)
$= \text{P(T)} \times \text{P(H)} \times \text{P(T)}$
$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$
$= \frac{1}{8}$

**3** Let $A$ be the event of photocopier A malfunctioning and $B$ be the event of photocopier B malfunctioning.

**a** P(both malfunction)
$= \text{P}(A \text{ and } B)$
$= \text{P}(A) \times \text{P}(B)$
$= \frac{8}{100} \times \frac{12}{100}$
$= \frac{96}{10\,000}$
$= \frac{6}{625}$

**b** P(both work)
$= \text{P}(A' \text{ and } B')$
$= \text{P}(A') \times \text{P}(B')$
$= \frac{92}{100} \times \frac{88}{100}$
$= \frac{8096}{10\,000}$
$= \frac{506}{625}$

**4** **a** P(they will be happy)
$= \text{P(B, then G, then B, then G)}$
$= \text{P(B)} \times \text{P(G)} \times \text{P(B)} \times \text{P(G)}$
$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$
$= \frac{1}{16}$

**b** P(they will be unhappy)
$= 1 - \text{P(they will be happy)}$
$= 1 - \frac{1}{16}$
$= \frac{15}{16}$

**5** Let $J$ be the event of Jiri hitting the target and $B$ be the event of Benita hitting the target.
$\therefore \quad \text{P}(J) = 0.7, \quad \text{P}(J') = 0.3, \quad \text{P}(B) = 0.8, \quad \text{P}(B') = 0.2$

**a** P(both hit)
$= \text{P}(J) \times \text{P}(B)$
$= 0.7 \times 0.8$
$= 0.56$

**b** P(both miss)
$= \text{P}(J') \times \text{P}(B')$
$= 0.3 \times 0.2$
$= 0.06$

**c** P(J hits and B misses)
$= \text{P}(J) \times \text{P}(B')$
$= 0.7 \times 0.2$
$= 0.14$

**d** P(B hits and J misses)
$= \text{P}(B) \times \text{P}(J')$
$= 0.8 \times 0.3$
$= 0.24$

**6** Let $H$ be the event the archer hits the bullseye. $\therefore \quad \text{P}(H) = \frac{2}{5}, \quad \text{P}(H') = \frac{3}{5}$

**a** P(3 hits)
$= \text{P}(H) \times \text{P}(H) \times \text{P}(H)$
$= \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$
$= \frac{8}{125}$

**b** P(2 hits then a miss)
$= \text{P}(H) \times \text{P}(H) \times \text{P}(H')$
$= \frac{2}{5} \times \frac{2}{5} \times \frac{3}{5}$
$= \frac{12}{125}$

**c** P(all misses)
$= \text{P}(H') \times \text{P}(H') \times \text{P}(H')$
$= \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$
$= \frac{27}{125}$

## EXERCISE 9D.2

**1** **a** P(all strawberry creams)
$= \text{P(1st is S } \textit{and}\text{ 2nd is S } \textit{and}\text{ 3rd is S)}$
$= \frac{8}{12} \times \frac{7}{11} \times \frac{6}{10}$
$= \frac{14}{55}$

**b** P(none are strawberry creams)
$= \text{P(1st is S}' \textit{and}\text{ 2nd is S}' \textit{and}\text{ 3rd is S}')$
$= \frac{4}{12} \times \frac{3}{11} \times \frac{2}{10}$
$= \frac{1}{55}$

**2** **a** P(both red)
$= \text{P(1st is R } \textit{and}\text{ 2nd is R)}$
$= \frac{7}{10} \times \frac{6}{9}$
$= \frac{7}{15}$

**b** P(GR)
$= \text{P(1st is G } \textit{and}\text{ 2nd is R)}$
$= \frac{3}{10} \times \frac{7}{9}$
$= \frac{7}{30}$